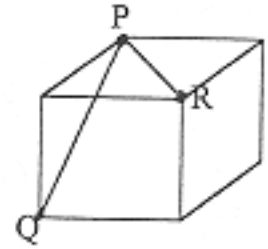
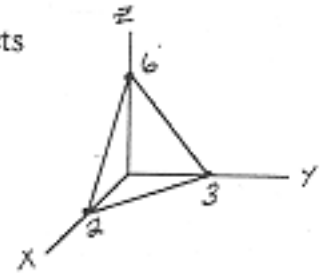
SM221 Calculus III Chapter 9 Practice Test Fall 1999

1. A 1x1x1 cube has corners P, Q, and R as shown on the right. Use vectors to find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.



2. a) Find an equation for the plane on the right which intersects the x-axis at 2, the y-axis at 3, and the z-axis at 6.
   b) What is the area of the triangle shown?
   c) How far is the plane from the origin?



3. Let $\vec{v} = <1, 3>$ and $\vec{w} = <4, 2>$.
   a) Find the scalar $comp_{\vec{w}}\,\vec{v}$. b) Find the vector $proj_{\vec{w}}\,\vec{v}$.
   c) Find the vector projection of $\vec{v}$ perpendicular to $\vec{w}$ (i.e. $orth_{\vec{w}}\,\vec{v}$).
   d) Draw $\vec{v}$, $\vec{w}$, $proj_{\vec{w}}\,\vec{v}$, and $orth_{\vec{w}}\,\vec{v}$ on the same set of axes.

4. a) Find a point onboth planes $x + y + z = 3$ and $2x - y + z = 1$. (Hint: set $x = 0$.)
   b) Find the line where the two planes in a) intersect. (Hint: The normal vectors to the planes are both perpendicular to the line.)
   c) Where does your line from b) intersect the x-y coordinate plane?

5. Sketch the graphs for the functions (use traces if necessary):
   a) $f(x,y) = \cos(y)$ b) $f(x,y) = x^2$ c) $f(x,y) = 4x^2 + y^2$

6. a) The equation for a surface in cylindrical coordinates is $z = r^2$. Rewrite the equation in rectangular coordinates and sketch it.
   b) Sketch the solid whose spherical coordinates satisfy the inequalities: $0 \le \phi \le \pi/6$ and $0 \le \rho \le 2$.

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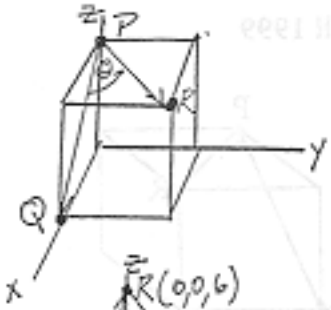
1. 

The points P, Q, and R have coordinates (0,0,1), (1,0,0) and (1,1,1) respectively. So $\overrightarrow{PQ} = \langle 1,0,-1\rangle$, $\overrightarrow{PR} = \langle 1,1,0\rangle$.

$\overrightarrow{PQ}\cdot\overrightarrow{PR} = |\overrightarrow{PQ}||\overrightarrow{PR}|\cos\theta$

$1 = \sqrt{2}\sqrt{2}\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} = 60°.$

(an easy non-vector method: ΔPQR is equilateral $\Rightarrow \theta = 60°$)

2.

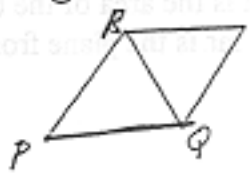
a) normal vector $\vec{N} = \overrightarrow{PQ}\times\overrightarrow{PR} = \langle -2,3,0\rangle \times \langle -2,0,6\rangle$

$= \begin{vmatrix} i & j & k \\ -2 & 3 & 0 \\ -2 & 0 & 6 \end{vmatrix} = \langle 18,12,6\rangle. \Rightarrow 18(x-2)+12(y-0)+6(z-0)=0$

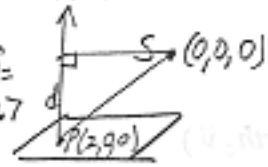
or $3x+2y+z=6$ or $\frac{x}{2}+\frac{y}{3}+\frac{z}{6}=1.$

b) area of triangle $= \frac{1}{2}$ area of parallelogram

$= \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}| = \frac{1}{2}|\langle 18,12,6\rangle| = 3\sqrt{14}$



c) $\vec{N} = \langle 18,12,6\rangle$

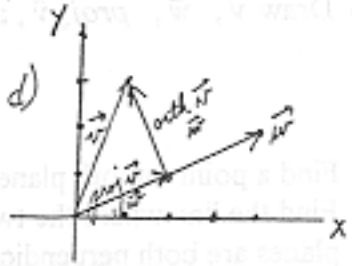


$d = \left|\text{comp}_{\vec{N}}\overrightarrow{PS}\right| = \left|\overrightarrow{PS}\cdot\frac{\vec{N}}{|\vec{N}|}\right| = \left|\langle -2,0,0\rangle\cdot\frac{\langle 18,12,6\rangle}{3\sqrt{14}}\right| = \frac{12}{\sqrt{14}}.$

3. a) $\text{comp}_{\vec{w}}\vec{v} = \vec{v}\cdot\frac{\vec{w}}{|\vec{w}|} = \frac{\langle 1,3\rangle\cdot\langle 4,2\rangle}{\sqrt{20}} = \frac{10}{\sqrt{20}} = \frac{5}{\sqrt{5}}.$

b) $\text{proj}_{\vec{w}}\vec{v} = \left(\frac{5}{\sqrt{5}}\right)\frac{\vec{w}}{|\vec{w}|} = \frac{5}{\sqrt{5}}\frac{\langle 4,2\rangle}{\sqrt{20}} = \frac{1}{2}\langle 4,2\rangle = \langle 2,1\rangle.$

c) $\text{orth}_{\vec{w}}\vec{v} = \vec{v} - \text{proj}_{\vec{w}}\vec{v} = \langle 1,3\rangle - \langle 2,1\rangle = \langle -1,2\rangle$

d) 

4. a) let $x=0$: $y+z=3$ and $-y+z=1 \Rightarrow 2z=4 \Rightarrow z=2, y=1, x=0$

so $(0,1,2)$ is a point in both planes.

b)

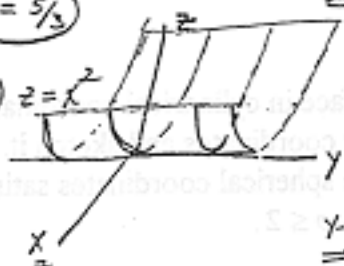
The line is $\perp$ both normals for the planes so $\vec{v} = \vec{N}_1\times\vec{N}_2$

$= \langle 1,1,1\rangle\times\langle 2,-1,1\rangle = \langle 2,1,-3\rangle \Rightarrow$ $x = 0+2t$, $y = 1+1t$, $z = 2-3t$.

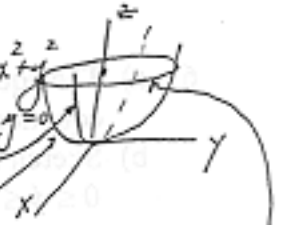
c) $z=0 \Rightarrow t = 2/3 \Rightarrow x = 4/3, y = 5/3$

5. a) $z = \cos y$

b) $z = x^2$



c) $z = 4x^2+y^2$



x-z trace: $y=0$ $\Rightarrow z = 4x^2$

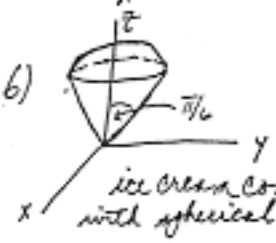
y-z trace: $x=0$ $\Rightarrow z = y^2$

trace on $z=4$ plane: $\Rightarrow 4 = 4x^2+y^2$ $\Rightarrow x^2+\frac{y^2}{4}=1$ ellipse

$\Rightarrow$ an elliptic paraboloid

6. a) $z = r^2 = x^2+y^2$

b)



ice cream cone with spherical top